Benha University
Faculty Of Engineering at Shoubra

ECE 122
Electrical Circuits (2)(2016/2017)
Lecture (4)
Parallel Resonance (P.2)

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Remember

Parallel Resonance Circuit

It is usually called tank circuit

Ideal Circuits

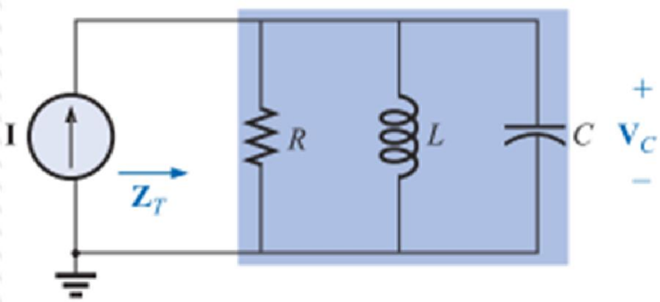


FIG. 20.21
Ideal parallel resonant network.

Practical Circuits

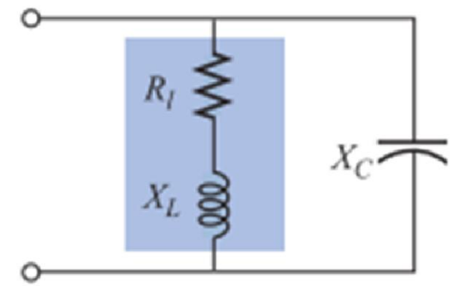
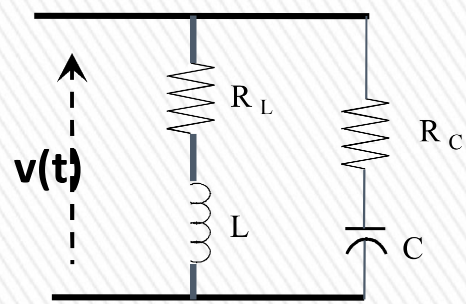


FIG. 20.22
Practical parallel L-C network.

Complex Configuration



Practical Parallel Resonance Circuit

Effect of Winding Resistance on the Parallel Resonant Frequency

- The internal resistance of the coil must be taken into consideration because it is no longer be included in a simple series or parallel combination with the source resistance and any other resistance added for design purposes.
- Even though R_L is usually relatively small in magnitude compared with other resistance and reactance levels of the network, it does have an important impact on the parallel resonant condition,

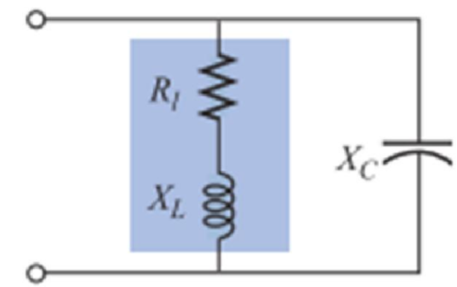


FIG. 20.22
Practical parallel L-C network.

1. Find a parallel network equivalent to the series R-L branch

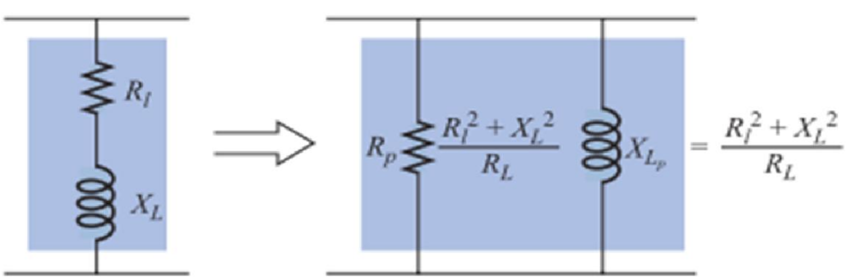


FIG. 20.23
Equivalent parallel network for a series R-L combination.

$$\mathbf{Z}_{R\text{-}L} = R_l + j X_L$$

$$\mathbf{Y}_{R\text{-}L} = \frac{1}{\mathbf{Z}_{R\text{-}L}} = \frac{1}{R_l + j X_L} = \frac{R_l}{R_l^2 + X_L^2} - j \frac{X_L}{R_l^2 + X_L^2}$$

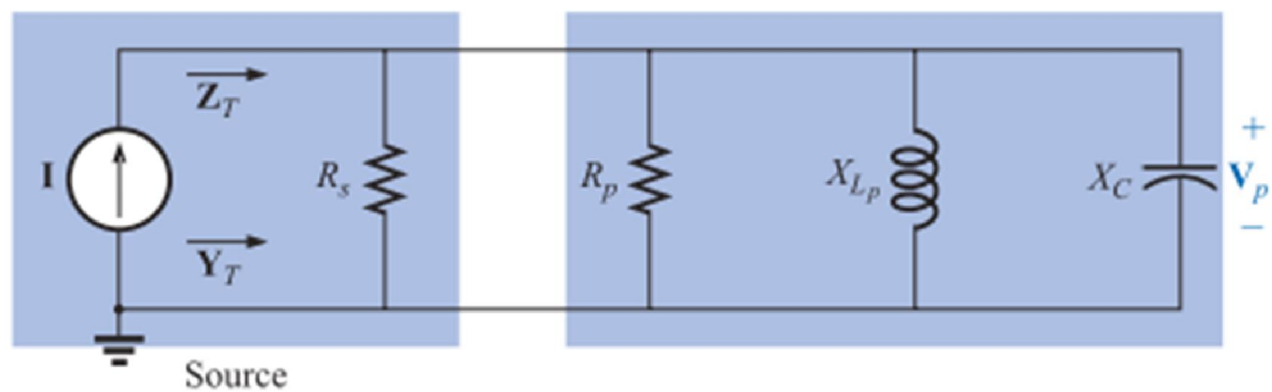
Practical Parallel Resonance Circuit

$$\mathbf{Y}_{R\text{-}L} = \frac{1}{\dfrac{R_l^2 + X_L^2}{R_l}} + \frac{1}{j\left(\dfrac{R_l^2 + X_L^2}{X_L}\right)} = \frac{1}{R_p} + \frac{1}{j\,X_{Lp}}$$

$$R_p = \frac{R_l^2 + X_L^2}{R_l}$$

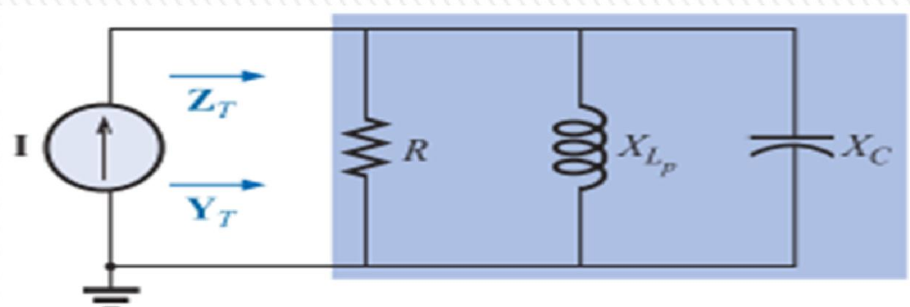
$$X_{L_p} = \frac{R_l^2 + X_L^2}{X_L}$$

Redrawing the network



If we define the parallel combination of R_s and R_p by the notation

$$R = R_s \parallel R_p$$



Practical Parallel Resonance Circuit

$$\mathbf{Y}_T = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_{L_p}}\right)$$

$$\frac{1}{X_C} - \frac{1}{X_{L_p}} = 0$$

$$\frac{1}{X_C} = \frac{1}{X_{L_p}}$$

$$X_{L_p} = X_C$$

$$\frac{R_l^2 + X_L^2}{X_L} = X_C$$

The resonant frequency, fp , can now be determined as follows:

$$R_l^2 + X_L^2 = X_C X_L = \left(\frac{1}{\omega C}\right)\omega L = \frac{L}{C}$$

$$X_L^2 = \frac{L}{C} - R_l^2$$

$$2\pi f_p L = \sqrt{\frac{L}{C} - R_l^2}$$

$$f_p = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R_l^2}$$

Multiplying within the square-root sign by C/L and rearranging produces :

$$f_p = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R_l^2 C}{L}}$$

$$f_p = f_s\sqrt{1 - \frac{R_l^2 C}{L}}$$

Practical Parallel Resonance Circuit

1. Maximum impedance

- At f = fp the input impedance of a parallel resonant circuit will be near its maximum value but not quite its maximum value due to the frequency dependence of Rp .

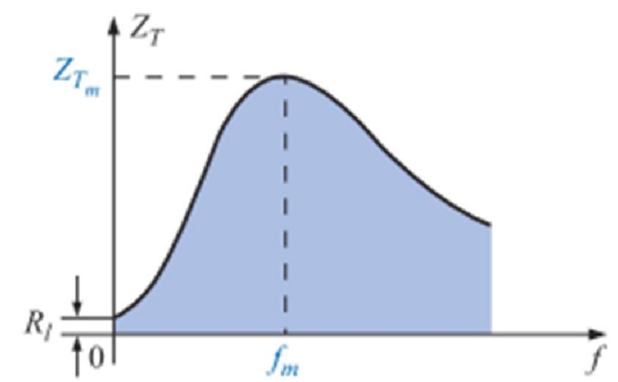


FIG. 20.26
Z_T versus frequency for the parallel resonant circuit.

- The frequency at which maximum impedance will occur is:

$$f_m = f_s\sqrt{1 - \frac{1}{4}\left(\frac{R_l^2 C}{L}\right)}$$

fm is determined by differentiating the general equation for ZT with respect to frequency

2. Minimum impedance

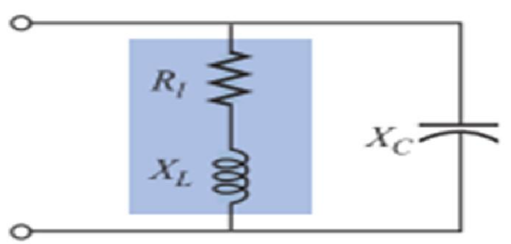


FIG. 20.22
Practical parallel L-C network.

At f = 0 Hz,

Xc is O.C, XL = zero

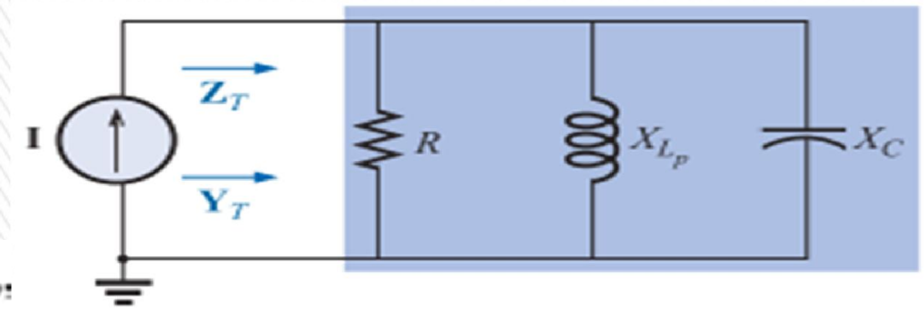
$$Z_T = R_s \parallel R_l \cong R_l.$$

As Rs is sufficiently large for the current source (ideally infinity)

Practical Parallel Resonance Circuit

➢ The quality factor of the practical parallel resonant circuit

determined by the ratio of the reactive power to the real power at resonance



$$Q_p = \frac{V_p^2/X_{L_p}}{V_p^2/R}$$

$$R = R_s \parallel R_p:$$

V_p is the voltage across the parallel branches.

$$Q_p = \frac{R}{X_{L_p}} = \frac{R_s \parallel R_p}{X_{L_p}}$$

$$Q_p = \frac{R_s \parallel R_p}{X_C}$$

For the ideal current source ($R_s = \infty\ \Omega$)

$$R = R_s \parallel R_p \cong R_p$$

$$Q_p = \frac{R_s \parallel R_p}{X_{L_p}} = \frac{R_p}{X_{L_p}} = \frac{(R_l^2 + X_L^2)/R_l}{(R_l^2 + X_L^2)/X_L}$$

$$Q_p = \frac{X_L}{R_l} = Q_l$$

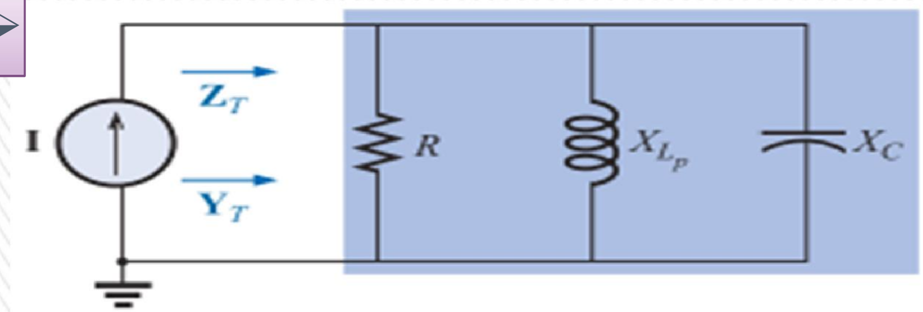
$R_s \gg R_p$

which is simply the quality factor Q_l of the coil.

Practical Parallel Resonance Circuit

Bandwidth and Half-Power point ➢

$$BW = f_2 - f_1 = \frac{f_r}{Q_p}$$



- ➢ The cutoff frequencies f1 and f2 can be determined using the equivalent network shown in the figure:

$$\mathbf{Z} = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)} = 0.707R$$

$$f_1 = \frac{1}{4\pi C}\left[\frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]$$

$$f_2 = \frac{1}{4\pi C}\left[\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]$$

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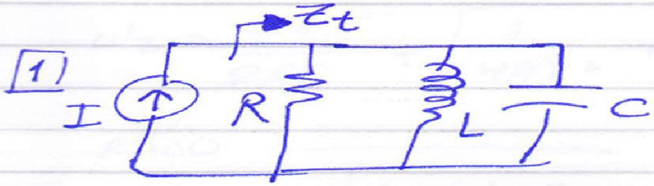
Parallel Resonance

Ideal circuit

[1]



[2] $X_L = X_C$ at Resonance

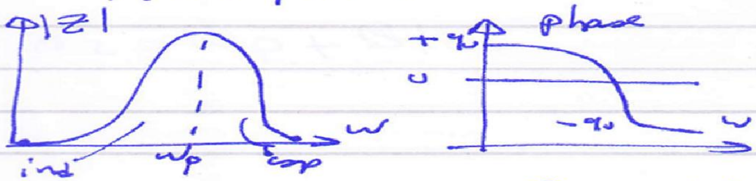
$\therefore \omega_p = \frac{1}{\sqrt{LC}}$ rad/s

[3] $Y = \frac{1}{Z} = Y_1 + Y_2 + Y_3$

$= \frac{1}{R} + j(\omega C - \frac{1}{\omega L})$

[4] at Resonance $G = \frac{1}{R}$ (← admittance)

& V, I inphase.



[5] $Q_p = \frac{R}{X_C} = \frac{R}{X_L} = \frac{\omega_p}{BW}$

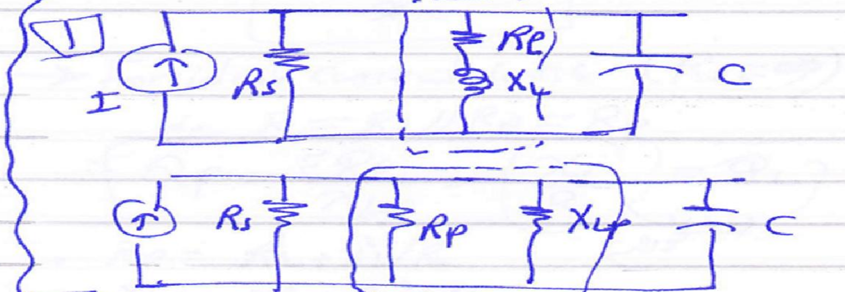
$= R/\omega L = \omega RC$

[6] $BW = \omega_2 - \omega_1 = \frac{1}{RC}$ rad/s

$= \omega_p / Q_p$

Practical circuit

[1]



[2] $Z = R_l + jX_L$, $Y = \frac{1}{Z} = \square + j\square$

$R_p = \frac{R_l^2 + X_L^2}{R_l}$ $= Q_l^2 R_l$ & $X_{Lp} = \frac{R_l^2 + X_L^2}{X_L}$ $= X_L$

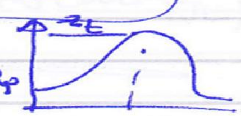
[3] $R = R_s \parallel R_p$

$Y_t = \frac{1}{R} + j(\frac{1}{X_C} - \frac{1}{X_{Lp}})$

[4] at Resonance $\frac{1}{X_C} = \frac{1}{X_{Lp}}$

$\therefore f_p = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{R_l^2 C}{L}}$

($\frac{1}{2\pi\sqrt{LC}}$ → $f_0 = f_s$)



[5] max impedance freq

$f_m \Rightarrow \frac{dZ_t}{df} = 0$

$f_m = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{1}{4}\frac{R_l^2 C}{L}}$

* Max impedance at $f = 0$ Hz

$X_L = 0$, X_C OC

$\therefore Z_t = R_s \parallel R_l \approx R_l$

[7] $$\omega_1 = \frac{-1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}} \quad rad/s$$

$$\omega_2 = \frac{+1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}} \quad rad/s$$

Also

[8] $$\omega_1 = \omega_p \sqrt{1 + \left(\frac{1}{2Q}\right)^2} - \omega_0/2Q$$

$$\omega_2 = \omega_p \sqrt{1 + \left(\frac{1}{2Q}\right)^2} + \omega_0/2Q$$

Note for Midband

$$\omega_1 = \omega_p - B/2$$

$$\omega_2 = \omega_p + B/2$$

[6] $$Q_p = \frac{R}{X_{Lp}} = \frac{R_s // R_p}{X_{Lp}}$$

$$= \frac{R_s // R_p}{X_c}$$

→ For ideal current source ($R_s \approx \infty$)

$\therefore R = R_s // R_p \simeq R_p$

$$\therefore Q_p = \frac{\simeq R_p}{X_{Lp}} = \frac{X_L}{R_L} = Q_L$$

[illegible]

→ $R_p = R_L^2 + X_L^2 / R_L$

→ $X_{Lp} = X_L^2 + R_p^2 / X_L$

[7] $$BW = f_r / Q_p = f_2 - f_1$$

$$= \omega_p / Q_p \quad \omega_p / Q_p = \frac{1}{RC} \quad rad/s$$

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}}$$

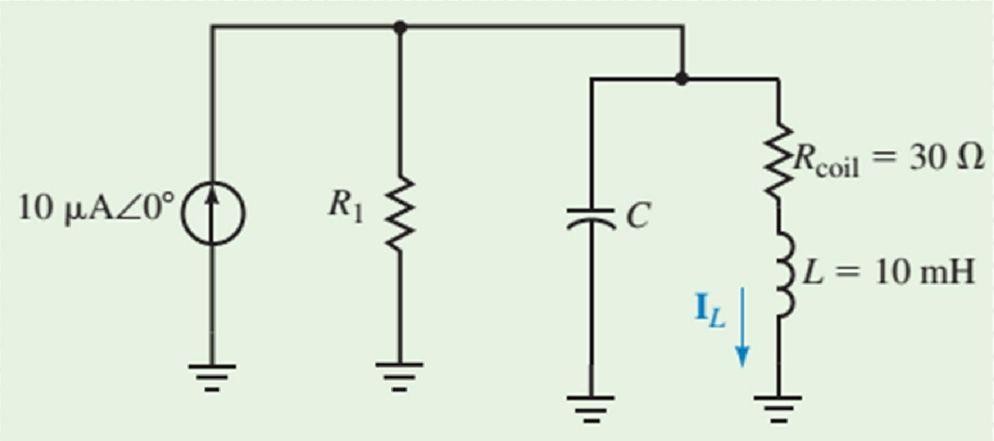
$$\omega_2 = +\frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}}$$

[8] $$Z = \frac{1}{\sqrt{2}} R = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)}$$

Examples

Example (1)

Determine the values of R1and C for the resonant tank circuit of the Figure so that the given conditions are met.
L=10 mH, R_{coil}=30Ω, f_P=58 kHz, BW =1 kHz, Solve for the current, I_L, through the inductor.



$$Q_P = \frac{f_P}{\text{BW(Hz)}} = \frac{58\text{ kHz}}{1\text{ kHz}} = 58$$

$$\omega_P = 2\pi f_P = (2\pi)(58\text{ kHz}) = 364.4\text{ krad/s}$$

$$C = \frac{1}{\omega_P^2 L} = \frac{1}{(364.4\text{ krad/s})^2(10\text{ mH})} = 753\text{ pF}$$

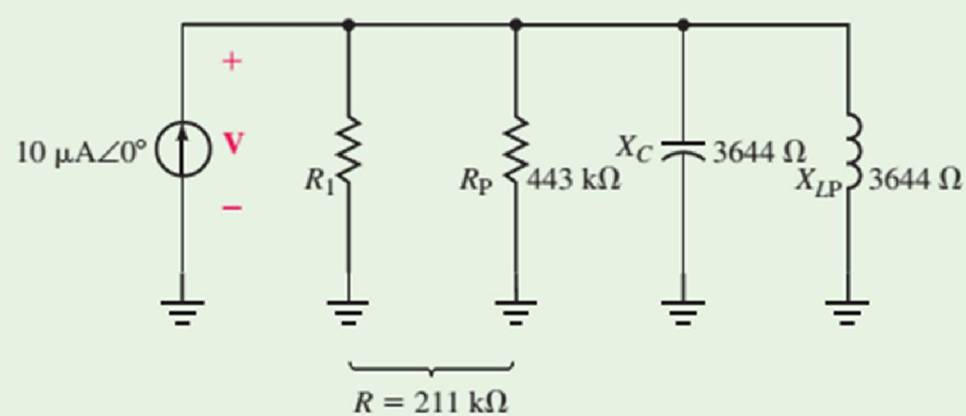
$$Q_{coil} = \frac{\omega_P L}{R_{coil}}$$

$$= \frac{(364.4\text{ krad/s})(10\text{ mH})}{30\ \Omega}$$

$$= \frac{3.644\text{ k}\Omega}{30\ \Omega} = 121.5$$

$$R_P \cong Q_{coil}^2 R_{coil} = (121.5)^2(30\ \Omega) = 443\text{ k}\Omega$$

$$X_{LP} \cong X_L = 3644\ \Omega$$



The quality factor Q_P is used to determine the total resistance of the circuit

$$R = Q_P X_C = (58)(3.644\ \text{k}\Omega) = 211\ \text{k}\Omega$$

But

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_P}$$

$$\frac{1}{R_1} = \frac{1}{R} - \frac{1}{R_P} = \frac{1}{211\ \text{k}\Omega} - \frac{1}{443\ \text{k}\Omega} = 2.47\ \mu\text{S}$$

And so

$$R_1 = 405\ \text{k}\Omega$$

The voltage across the circuit is determined to be

$$\mathbf{V} = \mathbf{I}R = (10\ \mu\text{A}\angle 0^\circ)(211\ \text{k}\Omega) = 2.11\ \text{V}\angle 0^\circ$$

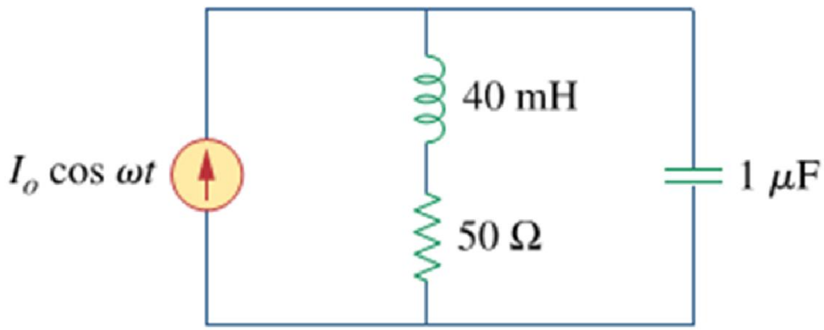
and the current through the inductor is

$$\mathbf{I}_L = \frac{\mathbf{V}}{R_{\text{coil}} + jX_L}$$

$$= \frac{2.11\ \text{V}\angle 0^\circ}{30 + j3644\ \Omega} = \frac{2.11\ \text{V}\angle 0^\circ}{3644\ \Omega\angle 89.95^\circ} = 579\ \mu\text{A}\angle -89.95^\circ$$

Example (2)

For the “tank” circuit in Fig., find the resonant frequency



$$Y = \frac{1}{R + j\omega L} + j\omega C = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

At resonance, $\text{Im}(\mathbf{Y}) = 0$, i.e.

$$\omega_0 C - \frac{\omega_0 L}{R^2 + \omega_0^2 L^2} = 0$$

$$R^2 + \omega_0^2 L^2 = \frac{L}{C}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \sqrt{\frac{1}{(40\times10^{-3})(1\times10^{-6})} - \left(\frac{50}{40\times10^{-3}}\right)^2}$$

$$\omega_0 = \underline{\mathbf{4841\ rad/s}}$$

Thank You